Chapter

3.1

Geometric Optics

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3.1.1 Introduction

Geometric optics deals with image formation using geometric methods. It is based on two postulates:

- That light travels in straight lines in a homogeneous medium
- That two rays may intersect without affecting the subsequent path of either

The fundamental laws of geometric optics can be developed from general principles, such as Maxwell's electromagnetic equations or Fermat's principle of least time. However, the laws of reflection and refraction can also be determined in a simple way by means of Huygen's principle, which states that every point of a wave front may be considered as a source of small waves spreading out in all directions from their centers, to form the new wave front along their envelope.

3.1.2 Laws of Reflection and Refraction

The laws of reflection and refraction for optics may be stated as follows:

- *Law of Reflection*. The angle of the reflected ray is equal to the angle of the incident ray.
- *Law of Refraction*. A ray entering a medium in which the velocity of light is different is refracted so that $n \sin i = n' \sin r$, where i is the angle of incidence, r is the angle of refraction, and n and n' are the indexes of refraction of the two media.

A *ray* is an imaginary line normal to the wave front. The angle the advancing ray forms with the line normal to the surface in question is the *angle of incidence* and is equal to the angle the wave front forms with the surface.

The *index of refraction* is the ratio of the velocity of light c in a vacuum to the velocity v in the medium

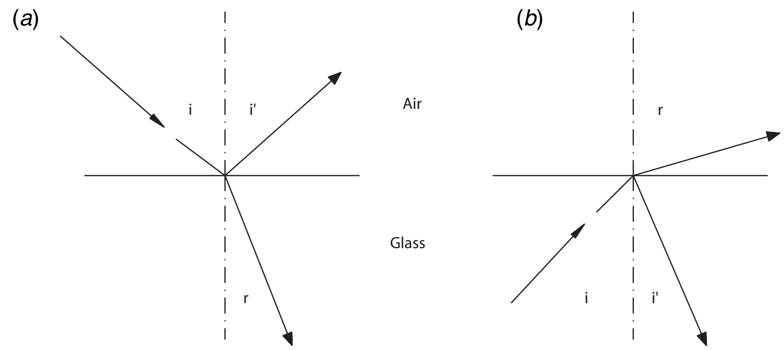

Figure 3.1.1 Refraction and reflection at air-glass surface: (*a*) beam incident upon glass from air, (*b*) beam incident upon air from glass. (*After* [1].)

$$n = \frac{c}{v} \qquad n' = \frac{c}{v'} \tag{3.1.1}$$

For air, the velocity of light is generally considered equal to the velocity *in vacuo* so $n = 1.0$ and the equation may be simplified to

$$\frac{\sin i}{\sin r} = n' \tag{3.1.2}$$

When a ray passes from a medium of smaller index into one of larger index, as from air to glass, the angle of refraction is less than the angle of incidence, and the ray is bent toward the normal. In passing from glass to air, the ray is bent away from the normal, as illustrated in Figure 3.1.1. The incident ray, reflected ray, refracted ray, and the normal to the surface at the point of incidence all lie in the same plane.

A ray passing from a medium of higher index to one of lower index may be totally internally reflected. The following relationship applies:

$$n \sin i = n' \sin r \tag{3.1.3}$$

The value of $\sin r$ is always greater than $\sin i$ when n is greater than n'. The maximum value for $\sin r$ is unity ($r = 90°$) and occurs for some value of i, called the *critical angle*, which is determined by the refractive indexes of the two media. For a water-air surface

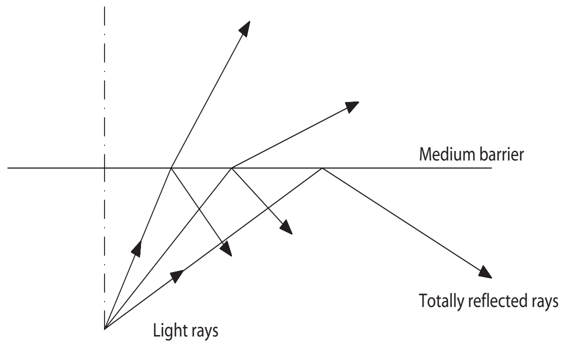

Figure 3.1.2 The result when the angle of refraction exceeds the critical angle and the ray is totally reflected. (*After* [1].)

$$\frac{n}{n'} = 1.33 \tag{3.1.4}$$

and the critical angle is 48.5°.

When the angle of incidence exceeds the critical angle, the ray is not refracted into the medium of lower index but is totally reflected, as illustrated in Figure 1.3.2. For angles smaller than the critical angle the rays are partially reflected.

Application of the *sine law* to two parallel surfaces, such as a glass plate, shows that the ray emerges parallel to the entering ray, but is displaced. The most important applications of the laws of reflection and refraction relate to the formation of images by means of spherical surfaces, such as mirrors and lenses.

3.1.2a Refraction at a Spherical Surface in a Thin Lens

It can be shown by tracing a ray through a single refracting surface that

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{R} \tag{3.1.5}$$

Where:
s = object distance to refracting surface
s' = image distance to refracting surface
R = radius of curvature of surface
n = index of refraction of object medium
n' = index of refraction of image medium

A ray traversing two refractive surfaces, as in a lens in air, has a path whose image distance and object distance are found by applying the foregoing equation to each of the two surfaces. For a lens whose thickness may be considered negligible relative to the image distance, the following applies:

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left\{\frac{1}{R_1} - \frac{1}{R_2}\right\} \qquad (3.1.6)$$

Where:
n = the index of refraction of the lens
R_1 = the radii of curvature of the first surface
R_2 = the radii of curvature of the second surface

The right side of the equation contains quantities that are characteristic of the lens, called the *power* of the lens. The reciprocal of this expression is referred to as the *focal length* f

$$\frac{1}{f} = (n-1)\left\{\frac{1}{R_1} - \frac{1}{R_2}\right\} \qquad (3.1.7)$$

For a thin lens in air the object distance, image distance, and focal length are related as follows:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad (3.1.8)$$

Certain conventions of algebraic sign must be observed in the use of this and previous equations. The conventions may be summarized as follows:

- All figures are drawn with the light incident on the reflecting or refracting surface from the left.
- The object distance s is considered positive where the object lies at the left of the *vertex*. The vertex is the intersection of the reflecting or refracting surface with the axis through the center of curvature of the surface.
- The image distance s' is considered positive when the image lies at the right of the vertex.
- The radii of curvature is considered positive when the center of curvature lies at the right of the vertex.
- Angles are considered positive when the slope of the ray with respect to the axis is positive.
- Dimensions, such as image height, are considered positive when measured upward from the axis.

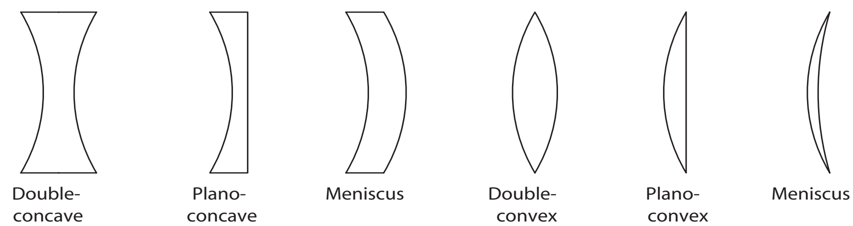

Figure 3.1.3 Various forms of simple converging and diverging lenses. (*After* [1].)

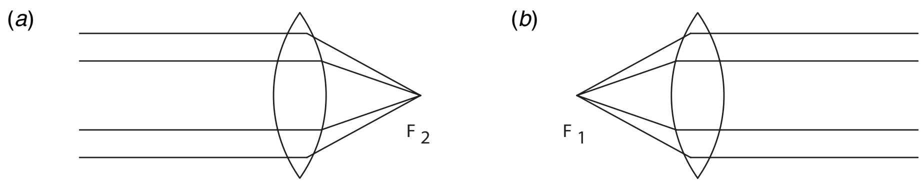

Figure 3.1.4 Lens effects: (*a*) parallel rays incident upon the lens pass through the second focal point, (*b*) rays passing through the first focal point incident upon the lens emerge parallel. (*After* [1].)

In general, after observing the first two conventions, the others follow the rules of coordinate geometry with the vertex as the origin.

From the previous equations and the foregoing sign conventions, it is apparent that the sign of the focal length may be negative or positive. For a lens in air, and parallel incident rays, the focal length is positive when the transmitted rays converge and negative when they diverge. Cross sections of simple converging and diverging lenses are shown in Figure 3.1.3.

There are two focal points of a lens, located on the lens axis. All incident rays parallel to the lens axis are refracted to pass through the second focal point; all incident rays from the first focal point emerge parallel to the lens axis, as illustrated in Figure 3.1.4. For a thin lens, the distances from the two focal points to the lens are equal and denote the focal length.

The magnification m provided by a lens is defined as the ratio of the image height (y') to the object height (y)

$$m = \frac{y'}{y} \tag{3.1.9}$$

The principles of magnification are illustrated in Figure 3.1.5. From the similar triangles *ABC* and *CDE*, it follows

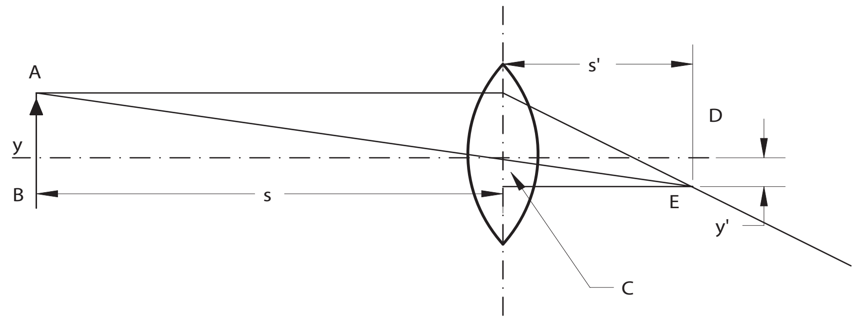

Figure 3.1.5 Magnification of a simple lens. (*After* [1].)

$$m = \frac{y'}{y} = \frac{s'}{s} \tag{3.1.10}$$

3.1.2b Reflection at a Spherical Surface

By considering reflection as a special case of refraction, many of the previous equations can be applied to reflection by a spherical mirror if the convention is adopted that $n' = -n$. This yields

$$\frac{1}{s} - \frac{1}{s'} = \frac{2}{R} \tag{3.1.11}$$

The focal point is the axial point that is imaged at infinity by the mirror; its distance from the mirror is the focal length. Hence

$$f = \frac{R}{2} \tag{3.1.12}$$

For a concave mirror, R is negative and the focal point lies at the left of the mirror, following the convention of signs described previously. For any spherical mirror

$$\frac{1}{s} - \frac{1}{s'} = \frac{1}{f} \tag{3.1.13}$$

It also follows that

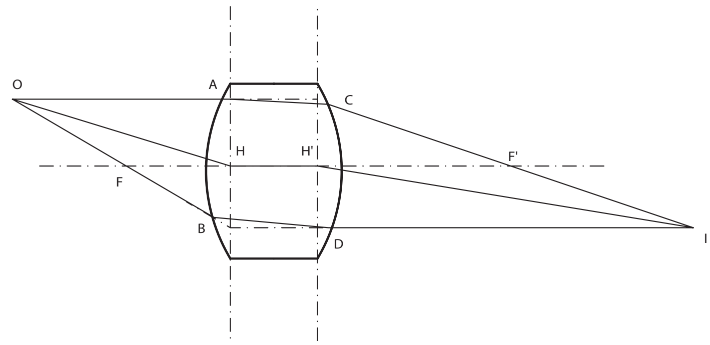

Figure 3.1.6 Focal points and principal planes for a thick lens. (*After* [1].)

$$m = \frac{y'}{y} = \frac{s'}{s} \tag{3.1.14}$$

Such mirrors are subject to spherical aberration as in lenses, such as the failure of the centrally reflected rays to converge at the same axial point as the rays reflected from the mirror edge. Aspherical surfaces formed by a *paraboloid of revolution* have the property that rays from infinity incident on the surface are all imaged at the same point on the axis. Thus, for the focal point and infinity, spherical aberration is eliminated. This is a useful device in projection components where the light source is placed at the focal point to secure a beam of nearly parallel rays.

Spherical aberration in mirrors can be eliminated by inserting lenses before the mirror. The *Schmidt corrector* is an aspherical lens, with one surface convex in the central region and concave in the outer region. The other surface is plane. A Schmidt system of spherical mirror and corrector plate can be made with a high relative aperture; $f/0.6$ is a typical value. Because of the efficiency and low cost of these systems, compared with projection lens systems, they have been used to obtain enlarged images from CRT devices for large screen display (among other applications).

Another type of corrector for a spherical mirror is a *meniscus lens* having no aspherical surfaces, known as a *Maksutov corrector*. The spherical surfaces of the meniscus lens permits easy manufacturing.

3.1.2c Thick (Compound) Lenses

The equations given previously apply to thin lenses. When the thickness of the lens cannot be ignored, measurements must be made from reference points other than the lens surface, such as the focal points—which have already been defined—or from the *principal points*. The principal points are located as follows (see Figure 3.1.6):

- Consider ray *OA* proceeding from the object parallel to the lens axis. This will be refracted to pass through the focal point F' .

- The ray *OB*, which passes through the focal point *F*, will emerge along *DI* parallel to the lens axis.
- If *OA* and *F′ I* are extended, their point of intersection lies in the second principal plane.
- The point *H′* , where this plane intersects the axis, is called the *second principal point*. Similarly, the intersection of *OF* and *DI* extended lies in the first principal plane, and *H* is the *first principal point*.
- The distances *FH* and *F′ H′* are the first and second focal lengths, respectively.

When the index of the medium on both sides of the lens is the same, as for a lens in air, the first and second focal lengths are equal.

If the direction of the light ray is reversed (the object is placed at the image position), the ray retraces its path and the image is formed at the former object position. Any two corresponding object and image points are said to be conjugate to each other, and hence are *conjugate points*.

The equation

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \tag{3.1.15}$$

given previously for a thin lens, continues to hold for a thick lens, but s and s' are measured from their respective principal points, as is the focal length f. The object distance, image distance, and focal length are related in another form, known as the *Newtonian form* of the lens equation. If x is the distance of the object from its focal point, and x' the image distance from its focal point, then

$$x\,x' = f^2 \tag{3.1.16}$$

3.1.2d Lens Aberrations

Up to this point, optical images have been considered to be faithful reproductions of the object. The equations given have been derived from the general expressions for the refraction of a ray at a spherical surface when the angle between the ray and the axis is small so that $\sin\theta = \theta$. This approximation is known as *first-order theory*. The departures of the actual image from the predictions of first-order theory are called aberrations. von Seidel extended the first-order theory by including the third-order terms of the expanded sine function. The *third-order theory* contains five terms to be applied to the first-order theory. When no aberrations are present, and monochromatic light is passed through the optical system, the sum of the five terms is zero. Thus von Seidel's sums provide a logical classification for the five monochromatic aberrations. In addition, two forms of chromatic aberration can occur because of variation of index with wavelength. The five monochromatic aberrations are:

- Spherical aberration
- Coma

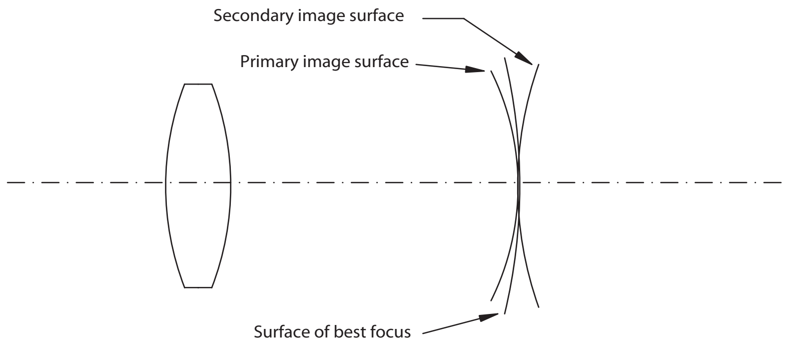

Figure 3.1.7 Surfaces of best focus, illustrating lens astigmatism. (*After* [1].)

- Astigmatism
- Curvature of field
- Distortion of field

Spherical aberration may be described as the failure of rays from an axial point to form a point image in the direction along the axis. In general, spherical aberration can be minimized if the deviation of the rays is equally divided between the front and rear surfaces of the lens. In a system of two or more lenses, spherical aberration can be eliminated by making the contribution of the negative elements equal and opposite to that of the positive elements.

Coma relates to failure of the rays from an off-axis point to converge at the same point in the plane perpendicular to the axis. Coma can be eliminated for a given object and image distance in a single lens by proper choice of radii of curvature.

Astigmatism contains aspects of both spherical aberration and coma. It resembles coma in that the off-axis points are affected, but—like spherical aberration—results from spreading of the image in a direction along the axis. The rays from a point converge on the other side of the lens to form a *line image*, actually the axis of a degenerate ellipse; continuing, the rays join with other rays to form a circle, and then at a still further distance form a second image crossed perpendicularly to the first. The best focus occurs when a circular image is formed. The locus of inner line images—the primary images—is a surface of revolution about the lens axis, called the *primary image surface*, shown in Figure 3.1.7. The locus of outer line images forms the secondary image surface. The locus of *circles of least confusion* forms the *surface of best focus*. As shown in the figure, these surfaces are tangent to one another at the lens axis.

Astigmatism is the failure of the primary and secondary image surfaces to coincide. The surface of best focus is usually not a plane but a curved surface; this type of aberration is known as *curvature of field*. It is not possible to eliminate both astigmatism and curvature of field in a single lens.

All rays passing through a lens from the center to the edge should result in equal magnification of the image. Distortion of the image occurs when the magnification varies with axial distance. If the magnification increases with axial distance, the effect is known as *pincushion*

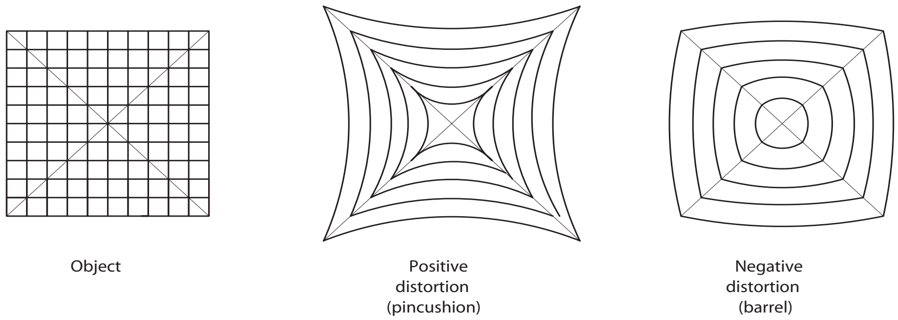

Figure 3.1.8 Pincushion and barrel distortion in the image of a lens system. (*After* [1].)

distortion, and the opposite effect is known as *barrel distortion*. The types of distortion are illustrated in Figure 3.1.8.

The five types of lens aberration described here can occur in uncorrected lenses even though light of a single wavelength forms the image. When the image is formed by light from different regions of the spectrum, two types of *chromatic aberration* can occur:

- *Axial* or *longitudinal chromatism*
- Lateral chromatism

Axial chromatism results from the convergence of rays of different wavelength at different points along the axis; the lens focal length varies with wavelength. Because magnification depends upon the focal length, the images are also of different size, producing lateral chromatism. In many instances, lenses are corrected so that the focal points coincide for two or three colors, thus eliminating longitudinal chromatism. However, unless the focal lengths are also made to coincide, the images will be of slightly different size. This defect results in color fringing in the outer portions of the field.

3.1.2e Lens Stops

It is obvious in the case of a simple lens that the rim of the lens forms the limiting boundary for rays transmitted by the lens. The introduction of smaller apertures before or after the lens can further limit the bundle of transmitted rays. This is done to eliminate unwanted rays that would produce distortions, to control the quantity of light transmitted, or to control the field of view. An aperture that controls the quantity of light transmitted, as the iris diaphragm in a camera, is called an *aperture stop*. (See Figure 3.1.9). An aperture that controls the field of view is called a *field stop.* The image of the aperture stop, projected into the object space, is called the *entrance pupil* of the lens system. The image of the aperture stop in the image space is called the *exit pupil.*

The relative aperture of a lens, usually called the *f*-number, is the ratio of the focal length to the effective lens diameter. A lens of *f*/3.5 has a focal length 3.5 times its effective diameter. In photographic objectives, the lens stop may be reduced from its maximum, rated value to a limiting value, usually *f*/22. Because the focal length remains constant, the effective area of the lens,

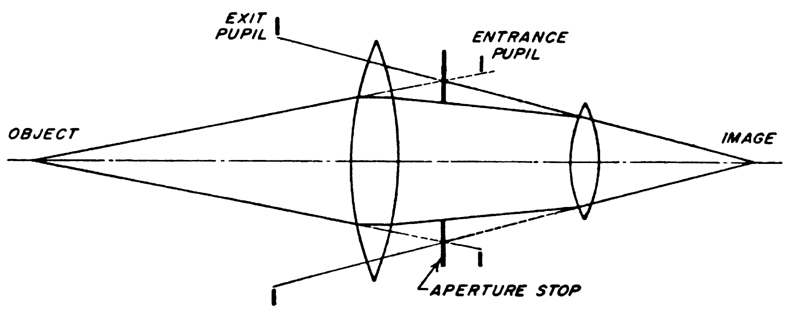

Figure 3.1.9 Aperture stop, entrance pupil, and exit pupil for a lens system. (*From* [1]. *Used with permission.*)

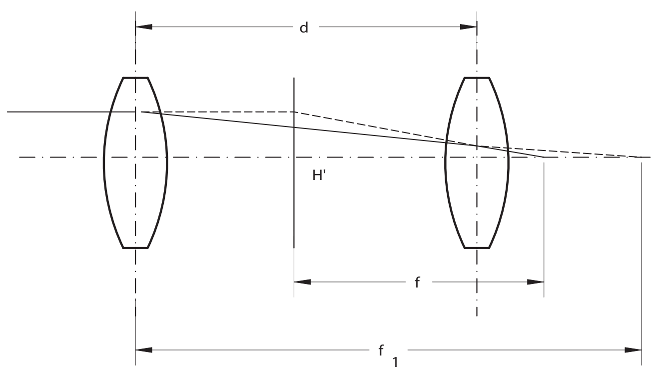

Figure 3.1.10 Lens system treated as a single thick lens. (*After* [1].)

and hence the amount of light transmitted, varies inversely as the square of the *f*-number. Thus, a lens set at *f*/8 passes nearly twice as much light as the same lens set at *f*/11.

3.1.3 Lens Systems

A combination of lenses may be treated as a thick lens. Consider two thin lenses of focal lengths f_1 and f_2 separated by a distance d. (See Figure 3.1.10.) The second principal plane is found in the same manner as for a single thick lens. The focal length of the combination f is the distance from the focal point to the principal plane. It is related to the focal lengths of the two thin lenses by

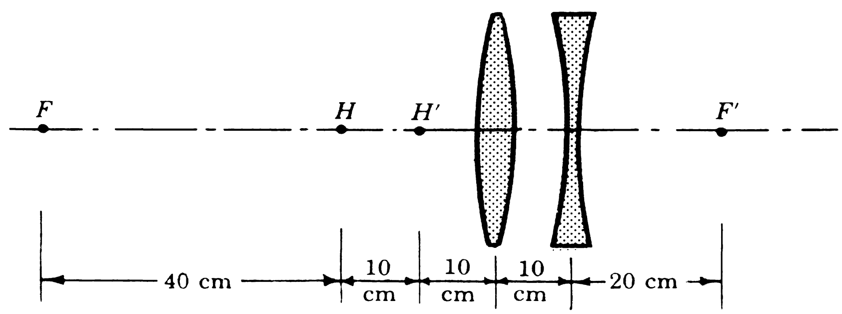

Figure 3.1.11 Principle of the telephoto lens.

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \tag{3.1.17}$$

This equation can also be applied to calculate the focal length of a typical telephoto lens system, shown simplified in Figure 3.1.11. If the positive lens has a focal length of +20 cm, the negative lens a focal length of –20 cm, and they are separated by a distance of 10 cm, Equation (3.1.17) shows that the focal length of the system is 40 cm. The system has a long focal length, but the rear-element-to-pickup-element distance is half the focal length.

Lenses are shaped to have *spherical properties* (uniform properties about the center of the lens) or *cylindrical properties* (uniform properties about the horizontal or vertical axis of the lens). An *anamorphic lens* is designed to produce different magnification of an image in the horizontal and vertical axes.

Anamorphic lenses are described in terms of their aspect ratio, the ratio of width to height of the screen image. The historical motion picture screen dimensions are 4 units wide to 3 units high, or an aspect ratio of 1.33:1. In 1953, the Cinemascope system was introduced, with an aspect ratio of 2.35:1 at the screen. Other motion picture aspect ratios include 1.65:1 and 1.85:1. The conventional television screen ratio uses the historical standard 1.33:1. High-definnition television (HDTV) typically uses a 1.77:1 (16:9) aspect ratio.

3.1.4 References

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Chapter

3.2

Fundamental Optical Elements

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3.2.1 Introduction

Cameras and many color projection display systems require spatial separation of the red, green, and blue source light. These beams may also need to be filtered to eliminate spurious or undesired wavelengths. The design goal of a color beam-splitting system is to reflect all the light of one primary color and to transmit the remaining visible radiation. *Dichroic mirrors* and prisms are used with supplemental trimming filters to accomplish this end. The most efficient systems utilize dichroic mirrors.

3.2.2 Color Beam-Splitting Systems

A dichroic mirror is made by coating glass with alternate layers of two materials having high and low indices of refraction. The material must have a thickness of 1/4-wavelength at the center of the band to be reflected. Figure 3.2.1 shows a typical mirror arrangement. The blue light is reflected by the first mirror, and the red and green light is transmitted. The red light is reflected by the second mirror, and the green light is passed. The curves of Figure 3.2.2 show typical transmittance versus wavelength characteristics. The blue reflecting mirror transmits about 90 percent of the green and red light, and the red reflecting mirror transmits nearly 90 percent of the blue and green light.

3.2.2a Dichroic Prism

It can be seen in Figure 3.2.3 that when the angle of incidence of a light ray exceeds the critical angle, the ray is totally reflected. The critical angle for an air-glass surface is 42° for a typical index of refraction for glass of 1.50. Hence, a 45-45-90° glass prism offers a totally reflecting surface. Other designs permit partial reflection and refraction. Coatings at the prism surface, as for dichroic mirrors, will selectively pass or reflect different colors.

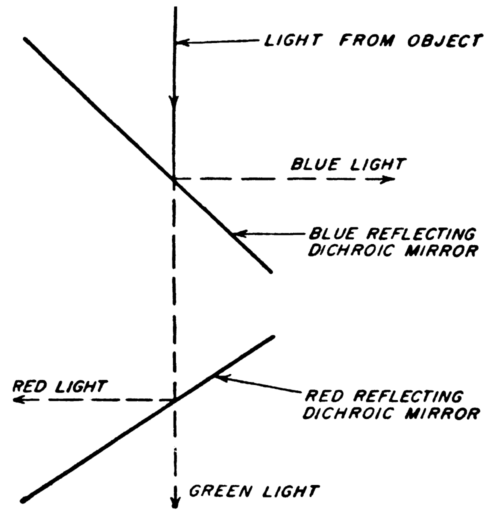

Figure 3.2.1 Arrangement of a dichroic mirror beam-splitting system. (*After* [1].)

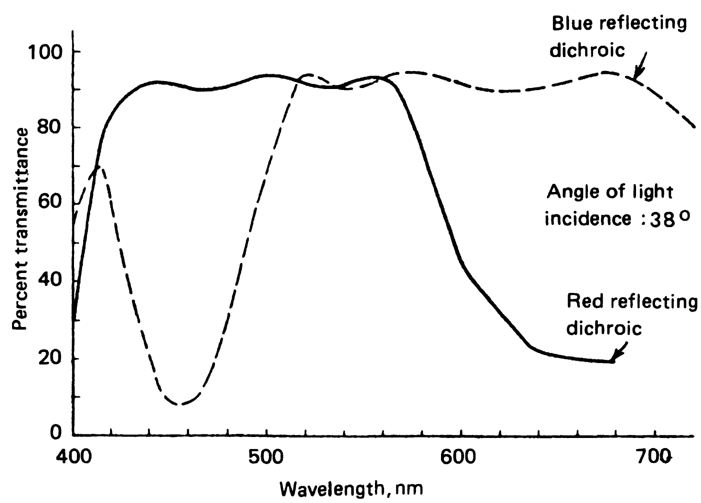

Figure 3.2.2 Transmission characteristics of typical dichroic mirrors.

3.2.2b Spectral Trim Filters

A typical dichroic mirror does not abruptly change spectral reflection at some specific wavelength. This property can be observed in Figure 3.2.2. Instead, there is a gradual transition over a wide band. This transition must be eliminated to maintain the purity of the red, green, and blue color signals. The spectral reflectance transmittance bands are trimmed by inserting filters hav-

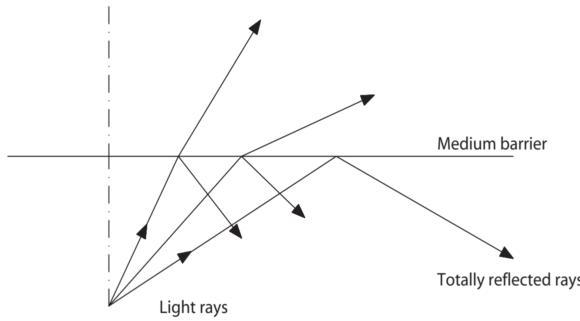

Figure 3.2.3 The result when the angle of refraction exceeds the critical angle and the ray is totally reflected. (*After* [1].)

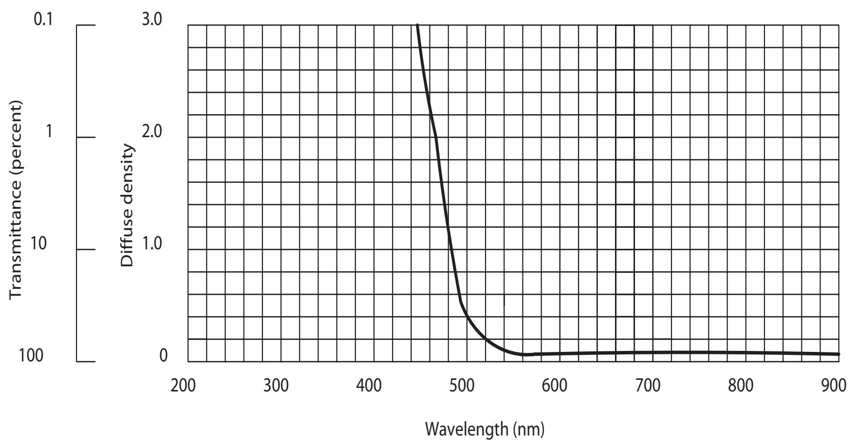

Figure 3.2.4 Response curve of a spectral trim filter (yellow). (*Source: Eastman Kodak Company.*)

ing abrupt divisions between high and low transmittance. These filters are constructed of glass, plastic, or gelatin containing light-absorbing substances.

The *neutral density filter* is another type useful in beam splitter applications. The filter absorbs equally, or nearly so, all wavelengths in the visible spectrum. These filters are available in different densities so that color beams can be balanced for equal signal output.

Figure 3.2.4 shows the response curve of a spectral trim filter (yellow). Figure 3.2.5 illustrates the response of a neutral density filter (D = 1.0).

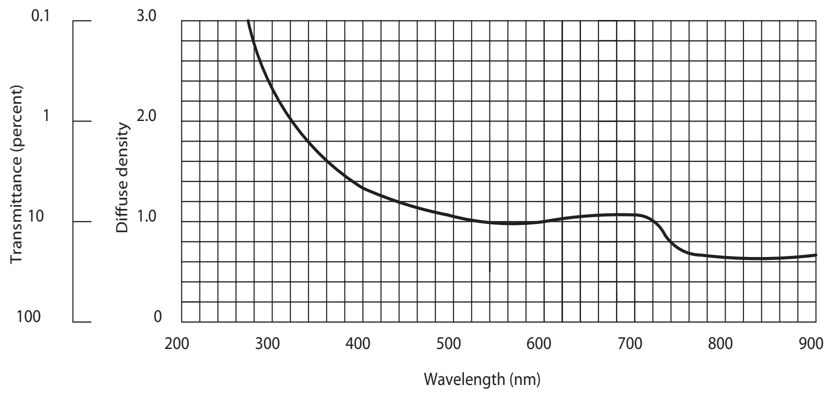

Figure 3.2.5 Response curve of a neutral density filter (visible spectrum). (*Source: Eastman Kodak Company.*)

3.2.3 Interference Effects

Huygen's principle was mentioned in Chapter 3.1 as forming the basis for the laws of reflection and refraction. To this concept should now be added the *principle of superposition*, which states that the resultant effect of the superposition of two or more waves at a point may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

If the wave path is thought of as a sinuous path consisting of alternate crests and troughs, the maximum height of the crests or depth of the troughs is called the *maximum amplitude*. Starting at zero amplitude and progressing through a crest back to zero and through a trough to zero constitutes one cycle; the distance traveled through the medium is one wavelength. The number of cycles per unit time is the frequency. The distance traveled per unit time is the velocity. Therefore, the velocity u is the product of the wavelength λ and the frequency v

$$u = v\,\lambda \tag{3.2.1}$$

If two waves meet in such a manner that the crests reinforce each other to produce the maximum possible amplitude, the waves are said to be in phase, but if the crest meets the trough to produce the minimum possible amplitude, they are said to be 180° out of phase. Thus, the phase expresses the distance between the crests of the waves. If two waves of the same amplitude, traveling in the same or opposite directions, are 180° out of phase, they are said to *completely interfere* and no disturbance is noted. If the phase, amplitude, or frequency of the waves is not the same, then the waves can reinforce at certain points and destroy at other points to produce an interference pattern.

A commonly observed example of interference is the array of colors seen in a thin film of oil on a wet pavement. Light waves are reflected from the front and rear surfaces of the film. When the thickness of the film is an odd number of quarter wavelengths, the light of that wavelength reflected from the front and back surfaces of the film reinforces itself and light is strongly reflected (assuming normal incidence). For an even number of quarter wavelengths, destructive interference occurs. Thus, from one area of film only blue-green light may be reflected while from another area of different thickness red light can be observed.

If—instead of a thin film of oil—two reflecting surfaces, such as two glass surfaces, are placed together but not in complete optical contact, an interference pattern is formed by light from the front and rear surfaces. Frequently the pattern takes the form of concentric rings, called *Newton's rings*.

Interference patterns are useful in grinding optical surfaces. The new surface may be tested by bringing it in contact with a surface of known curvature and noting the shape and separation of the fringes. By repeating the test at intervals, the new surface may be gradually worked to the desired precision.

3.2.3a Diffraction Effects

If an obstacle such as a slit or straight edge is placed in a beam of light—according to Huygen's principle—each point along the slit becomes a source for new wavelets. It can be shown that as these wavelets fan out beyond the obstacle they tend to reinforce or destroy each other in various regions, forming an interference pattern. As the wavelets fan out beyond the obstacle, the light "bends around" it, producing light areas in regions that would be dark if the light traveled only in straight lines. The effects produced by blocking part of a wave front to form interference patterns are called *diffraction effects*. If a wave front is incident on a circular opening such as a lens aperture, the diffraction pattern consists of a bright central disk surrounded by alternate dark and bright rings. The angle α formed at the lens by the diffraction circle is dependent upon the diameter of the lens opening D

$$\alpha = \frac{2.4\lambda}{D} \tag{3.2.2}$$

The *diffraction grating* is an important device utilizing diffraction principles. This element consists essentially of a large number of parallel slits of the same width spaced at regular intervals. Light passing through the slits is diffracted to form interference patterns. The waves will reinforce to form a maximum when the following condition is met

$$\sin\theta = \frac{n\lambda}{d} \tag{3.2.3}$$

Where:
θ = angle of deviation from the direction of incident light
d = distance between successive grating slits
n = an integer denoting order of the maximum

Some light will pass directly through the grating. This is called the *zero order*. The first maximum (assuming monochromatic light) lies beyond the zero order and is called the *first order*. The next maximum is the second order and so on. If white light is incident on the grating, the zero order is a white image followed by a first-order spectrum, then second-order, and so on. By proper ruling of the grating lines, a large proportion of the incident light can be directed into one of the first-order spectra. These gratings are used in many spectral-analysis instruments because of their high efficiency.

3.2.3b Polarization Effects

Because light is a series of electromagnetic waves, each wave can be separated into its electric (E) and magnetic (H) vectors, vibrating in planes at right angles to each other. A series of electromagnetic waves will have E vectors, for example, vibrating in all possible planes perpendicular to the direction of travel. By means of reflection, double refraction, or scattering, the waves can be sorted into two resultant components with their E vectors at right angles to each other. Each ray is said to be *plane-polarized*, that is, made up of waves vibrating in a single plane. If two rays with waves of equal amplitude are brought together, they can form elliptically, plane, or circularly polarized light depending upon whether:

- The phase difference between the vibrating waves lies between 0 and $\pi/2$ for elliptical polarization
- The phase is at 0 or π for plane polarization
- The phase is at $\pi/2$ for circular polarization.

The angle of incidence at which light reflected from a polished surface will be completely polarized is given by the equation known as *Brewster's law*

$$\tan\theta = \frac{n'}{n} \tag{3.2.4}$$

Where n' and n are the indexes of refraction of the two media.

For glass and air, $n' = 1.5$ and $n = 1$, the polarizing angle is 56°. Of the natural light incident at the polarizing angle, about 7.5 percent is reflected and is polarized with its vibration plane perpendicular to the plane of incidence. The rest of the light is transmitted and consists of a mixture of the light with a vibration plane parallel to the plane of incidence and the balance of the perpendicular component. By passing the mixture through successive sheets of glass stacked in a pile, more of the perpendicular component is removed at each reflection and the transmitted fraction consists of the parallel component.

The velocity of a light wave through many transparent crystalline materials is not the same in all directions. Because the ratio of the velocity of light in a medium to the velocity in a vacuum is the index of refraction, these materials have more than one index of refraction. When oriented in one position with respect to the direction of the incident ray, the crystal behaves normally and that direction is called the *optic axis* of the crystal. A ray incident on the crystal to form an angle with the optic axis is broken into two rays, one of which obeys the ordinary laws of refraction and is called the *ordinary ray*; the second ray is called the *extraordinary ray*. The two rays are plane-polarized in mutually perpendicular planes. By eliminating one of the rays, such doubly refracting materials can be used to obtain plane-polarized light. In some materials, one of the components is more strongly absorbed than the other. Crystals of iodoquinine sulfate are an example. The parallel orientation of layers of such crystals in plastic has been used to form polarizing filters.

Kerr discovered that some liquids become doubly refracting when an electric field is applied. The *Kerr effect* makes it possible to control the transmission of light by an electric field. A Kerr cell consists of a transparent cell containing a liquid such as nitrobenzene. The cell is placed between crossed polarizers. When an electric field is applied light is transmitted; it is cut off when the field is removed.

3.2.4 References

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3.2.5 Bibliography

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